

DESCRIPTION LOGICS FOR RELATIVE TERMINOLOGIES

OR

WHY THE BIGGEST CITY IS NOT A BIG THING

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Abstract. Context-sensitivity has been for long a subject of study in linguistics, logic and computer science. Recently the problem of representing and reasoning with contextual knowledge has been brought up in the research on the Semantic Web. In this paper we introduce a conservative extension to Description Logic, the formalism underlying Web Ontology Languages, supporting representation of ontologies containing *relative terms*, such as ‘big’ or ‘tall’, whose meaning depends on the selection of *comparison class* (context). The solution rests on introduction of modal-like operators in the language and an additional modal dimension in the semantics, which is built upon the standard object dimension of the Description Logic languages and whose states correspond to selected subsets of the object domain. We present the syntax and semantics of the extended language and elaborate on its representational and computational features.

1. Introduction

It is a commonplace observation that the same expressions might have different meanings when used in different contexts. A trivial example might be that of the concept `The_Biggest`. Figure 1 presents three snapshots of the same knowledge base that focus on different parts of the domain. The extension of the concept visibly varies across the three takes. Intuitively, there seem to be no contradiction in the fact that individual London is an instance of `The_Biggest`, when considered in the context of European cities,

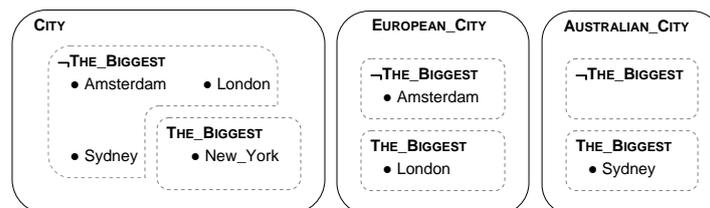


Figure 1: Example of a context-sensitive concept `The_Biggest`.

an instance of `¬The_Biggest`, when contrasted with all cities, and finally, not belonging to any of these when the focus is only on Australian cities. Natural language users resolve such superficial incoherencies simply by recognizing that certain terms, call them *relative*, such as `The_Biggest`, acquire definite meanings only when put in the context of specified *comparison classes* (Shapiro 2006, van Rooij to appear, Gaio 2008).

The problem of context-sensitivity has been for a long time a subject of studies in linguistics, logic and even computer science. Recently, it has been also encountered in the research on the Semantic Web (Bouquet, et al. 2003, Caliusco, et al. 2005, Benslimane, et al. 2006) where the need for representing and reasoning with imperfect information becomes ever more pressing (Lukasiewicz & Straccia 2008, Laskey, et al. 2008). Relativity of meaning appears as one of common types of such imperfection. Alas, *Description*

Logics (DLs), which form the foundation of the Web Ontology Language (Horrocks, et al. 2003), the basic knowledge representation formalism on the Semantic Web, were originally developed for modeling crisp, static and unambiguous knowledge, and as such, are incapable of handling the task seamlessly. Consequently, it has become clear that it is necessary to look for more expressive, ideally backward compatible languages to meet the new application requirements on the Semantic Web. Current proposals focus mostly on the problems of uncertainty and vagueness (Lukasiewicz & Straccia 2008, Straccia 2005), with several preliminary attempts of dealing with different aspects of contextualization of DL knowledge bases (Grossi 2007, Goczyla, et al. 2007, Benslimane et al. 2006). In this paper we propose a simple, conservative extension to the classical DLs, which is intended for representation of relative, context-sensitive terminologies, where by *contexts* we understand specifically the *comparison classes* with respect to which the terms acquire precise meanings.

To take a closer look at the problem consider again the scenario from Figure 1. On a quick analysis it should become apparent there is no straightforward way of modeling the scenario within the standard DL paradigm. Asserting both $\text{London} : \text{The_Biggest}$ and $\text{London} : \neg\text{The_Biggest}$ in the same knowledge base results in an immediate contradiction, which is obviously an unintended outcome. To avoid this consequence one can resort to the luring prospect of indexing, and instead assert $\text{London} : \text{The_Biggest}_{\text{European_City}}$ and $\text{London} : \neg\text{The_Biggest}_{\text{City}}$, with an implicit message that the two indexed concepts are meant to be two different ‘variants’ of *The_Biggest*, corresponding to two possible contexts of its use. The contradiction is indeed avoided, but unfortunately the baby has been thrown out with the bath water, for the two ‘variants’ become in fact two unrelated concept names, with no common syntactic or semantic core. More precisely, using this strategy one cannot impose global constraints on the contextualized concepts, for instance, to declare that regardless of the context, *The_Biggest* is always a subclass of *Big*. Even if this goal was achieved by rewriting constraints over all individual contexts, another source of problems is reasoning about the contexts themselves, for example, deciding whether an individual occurs in a given comparison class or not.

The extension proposed in this paper is, to our knowledge, unique in addressing this particular type of context-sensitivity in DL, and arguably, it cannot be simulated within any of the approaches present in the literature. Technically, the solution rests on the presence of special modal-like operators in the language and a second modal dimension in the semantics of the language, which is defined over the standard object dimension of DL and whose states correspond to selected subsets of the object domain. In the following section we formally define the language, next we elaborate on some of its basic representational and computational features, and finally, in the last two sections, we shortly position our work in a broader perspective and conclude the presentation.

2. Representation Language

We start by recalling preliminary notions regarding DLs and follow up with presentation of the syntax and the semantics of the proposed extension, for brevity denoted as DL^c .

2.1. Basic Description Logics

Description Logics are a family of knowledge representation formalisms, designed particularly for expressing terminological and factual knowledge about a domain of application (Baader, et al. 2003). For instance, the following DL formula defines the meaning of the concept European city by equating it to the set of all and only those individuals that are cities and are located in Europe; the next one asserts that New York is not in fact an instance of that concept:

$$\begin{aligned} \text{European_City} &\equiv \text{City} \sqcap \exists \text{located.in.}\{\text{Europe}\} \\ \text{New_York} &: \neg \text{European_City} \end{aligned}$$

Formally DLs can be seen as fragments of modal and first-order logic, with an intuitively appealing syntax (Schild 1991). A DL language \mathcal{L} is specified by the vocabulary $\Sigma = (N_I, N_C, N_R)$, consisting of a set of *individual names* N_I , *concept names* N_C , and *role names* N_R , and by a selection of logical operators Π (e.g.: $\sqsubseteq, \sqcap, \sqcup, \neg, \exists, \forall, \dots$), which allow for constructing complex expressions: concept descriptions, complex roles and axioms. A knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, expressed in \mathcal{L} , consists of the TBox \mathcal{T} , containing terminological constraints, (typically) of the form of inclusion $C \sqsubseteq D$ or equivalence axioms $C \equiv D$, for arbitrary concept descriptions C and D , and the ABox \mathcal{A} , including concept assertions $a : C$ and role assertions $(a, b) : r$, for individual names a, b , a concept C and a role r .

The semantics is defined in terms of an *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a non-empty *domain* of individuals, and $\cdot^{\mathcal{I}}$ is an *interpretation function*, which specifies the meaning of the vocabulary by mapping every $a \in N_I$ to an element of $\Delta^{\mathcal{I}}$, every $C \in N_C$ to a subset of $\Delta^{\mathcal{I}}$ and every $r \in N_R$ to a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The function is inductively extended over complex terms in a usual way, according to the fixed semantics of the logical operators. An interpretation \mathcal{I} *satisfies* an axiom in either of the following cases:

- $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- $\mathcal{I} \models a : C$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$
- $\mathcal{I} \models (a, b) : r$ iff $\langle a^{\mathcal{I}}, b^{\mathcal{I}} \rangle \in r^{\mathcal{I}}$

Finally, \mathcal{I} is said to be a *model* of a DL knowledge base, i.e. it makes the knowledge base true, if and only if it satisfies all its axioms.

2.2. Syntax

We consider an arbitrary DL language \mathcal{L} and extend it with special operators for expressing contextualized concept descriptions, roles and axioms.

Let C and D be concept descriptions in \mathcal{L} and r a role. Then the following are proper concept and role descriptions, respectively:

$$\begin{aligned} C, D &\rightarrow \diamond C \mid \square C \mid \langle D \rangle C \mid [D]C \\ r &\rightarrow \diamond r \mid \square r \mid \langle D \rangle r \mid [D]r \end{aligned}$$

The modal operators give access to the meaning of the bound terms in *specific subcontexts* of the current context of representation. The diamonds point at *certain* subcontexts, which might be either *anonymous* (\diamond -modality) or *named* by the qualifying concepts placed inside of the operators ($\langle \cdot \rangle$ -modality). In the former case the designated comparison class is unspecified, whereas in the latter it consists of all and only those individuals that, in the current context, are instances of the qualifying concept. Analogically, the dual box operators refer to *all* anonymous (\Box -modality), or named subcontexts ($[\cdot]$ -modality) of the current context.

For instance, $\langle \text{City} \rangle \text{The_Biggest}$ describes the individuals that are the biggest as considered in the context of cities, $\Box \neg \text{The_Biggest}$ refers to the individuals that are never *The_Biggest* regardless of the considered subcontext, while $\exists \langle \text{City} \rangle \text{nicer.European_City}$ uses the meaning of the role *nicer* as interpreted when talking about cities, and denotes those individuals which in this sense are nicer than some European cities.

In a similar manner we allow for contextualization of DL axioms. If α is a TBox/ABox axiom and D is a concept description, then the following are also proper TBox/ABox axioms in DL^c , respectively:

$$D, \alpha \rightarrow \quad \diamond \alpha \mid \Box \alpha \mid \langle D \rangle \alpha \mid [D] \alpha$$

For example, $\langle \text{Australian_City} \rangle (\text{Sidney} : \text{The_Biggest})$ asserts that there exists a context, namely that of Australian cities, in which Sidney is considered the biggest. The TBox axiom $\Box (\text{The_Biggest} \sqsubseteq \text{Big})$ enforces that regardless of the comparison class the concept *The_Biggest* is always subsumed by *Big*.

2.3. Semantics

The central semantic notion underlying DL^c is *context structure*, which can be seen as a special type of an interpretation of a *multi-dimensional DL* (Wolter & Zakharyashev 1999b).

Definition 1 (Context structure) A context structure over a DL^c language \mathcal{L} , with a set of operators Π and the vocabulary $\Sigma = (N_I, N_C, N_R)$, is a tuple $\mathcal{C} = \langle W, \triangleleft, \Delta, \{\mathcal{I}_w\}_{w \in W} \rangle$, where:

- $W \subseteq \wp(\Delta)$ is a set of possible contexts, such that $\Delta \in W$ and $\emptyset \notin W$;
- $\triangleleft \subseteq W \times W$ is an accessibility relation, such that for any $w, v \in W$, $w \triangleleft v$ if and only if $v \subseteq w$. In such cases we say that v is a subcontext of w ;
- Δ is a non-empty domain of interpretation;
- $\mathcal{I}_w = (\Delta^{\mathcal{I}_w}, \cdot^{\mathcal{I}_w})$ is a (partial) interpretation of \mathcal{L} in the context w :
 - $\Delta^{\mathcal{I}_w} = w$ is the domain of interpretation in w ,
 - $\cdot^{\mathcal{I}_w}$ is a standard interpretation function for language \mathcal{L}_w , defined by Π and a subset $\Sigma_w \subseteq \Sigma$ of the vocabulary of \mathcal{L} .

Note that the contexts are uniquely identifiable by their corresponding domains of interpretation and are ordered by \triangleleft according to the decreasing size of the domains, i.e. for every context structure and every $w, v \in W$ the following conditions hold:

- $w \triangleleft v$ iff $\Delta^{\mathcal{I}_v} \subseteq \Delta^{\mathcal{I}_w}$,
- if $\Delta^{\mathcal{I}_w} = \Delta^{\mathcal{I}_v}$ then $w = v$.

Finally, we observe there exists a special element $\hat{w} \in W$, denoted as the *top context*, such that $\Delta^{\mathcal{I}_{\hat{w}}} = \Delta$. Given the conditions above, it follows that \triangleleft imposes a partial order (reflexive, asymmetric and transitive) on the set of contexts, with \hat{w} as its least element. Thus context structures are built upon rooted partially ordered Kripke frames.

For an arbitrary context structure \mathcal{C} , a context w , concept descriptions C, D and a role r , the meaning of contextualized terms is inductively defined as follows:

$$\begin{aligned}
(\diamond C)^{\mathcal{I}_w} &= \{x \in \Delta^{\mathcal{I}_w} \mid \exists w \triangleleft v : x \in C^{\mathcal{I}_v}\} \\
(\square C)^{\mathcal{I}_w} &= \{x \in \Delta^{\mathcal{I}_w} \mid \forall w \triangleleft v : x \in \Delta^{\mathcal{I}_v} \rightarrow x \in C^{\mathcal{I}_v}\} \\
(\diamond r)^{\mathcal{I}_w} &= \{\langle x, y \rangle \in \Delta^{\mathcal{I}_w} \times \Delta^{\mathcal{I}_w} \mid \exists w \triangleleft v : \langle x, y \rangle \in r^{\mathcal{I}_v}\} \\
(\square r)^{\mathcal{I}_w} &= \{\langle x, y \rangle \in \Delta^{\mathcal{I}_w} \times \Delta^{\mathcal{I}_w} \mid \forall w \triangleleft v : x, y \in \Delta^{\mathcal{I}_v} \rightarrow \langle x, y \rangle \in r^{\mathcal{I}_v}\}
\end{aligned}$$

Terms contextualized via named contexts are interpreted analogically, except for an extra restriction imposed on the accessibility relation: only the subcontexts that match the extension of the qualifying concept are to be considered.

$$\begin{aligned}
(\langle D \rangle C)^{\mathcal{I}_w} &= \{x \in \Delta^{\mathcal{I}_w} \mid \exists w \triangleleft v, \Delta^{\mathcal{I}_v} = D^{\mathcal{I}_w} : x \in C^{\mathcal{I}_v}\} \\
([D]C)^{\mathcal{I}_w} &= \{x \in \Delta^{\mathcal{I}_w} \mid \forall w \triangleleft v, \Delta^{\mathcal{I}_v} = D^{\mathcal{I}_w} : x \in \Delta^{\mathcal{I}_v} \rightarrow x \in C^{\mathcal{I}_v}\} \\
(\langle D \rangle r)^{\mathcal{I}_w} &= \{\langle x, y \rangle \in \Delta^{\mathcal{I}_w} \times \Delta^{\mathcal{I}_w} \mid \exists w \triangleleft v, \Delta^{\mathcal{I}_v} = D^{\mathcal{I}_w} : \langle x, y \rangle \in r^{\mathcal{I}_v}\} \\
([D]r)^{\mathcal{I}_w} &= \{\langle x, y \rangle \in \Delta^{\mathcal{I}_w} \times \Delta^{\mathcal{I}_w} \mid \forall w \triangleleft v, \Delta^{\mathcal{I}_v} = D^{\mathcal{I}_w} : x, y \in \Delta^{\mathcal{I}_v} \rightarrow \langle x, y \rangle \in r^{\mathcal{I}_v}\}
\end{aligned}$$

Noticeably, the modalities involving named contexts nearly collapse, as there can always be only one such subcontext that matches the qualifying concept. Thus the inclusion $\square(\langle D \rangle C) \sqsubseteq [D]C$ is valid in $\text{DL}^{\mathcal{C}}$, although its converse $\square([D]C) \sqsubseteq \langle D \rangle C$ is not, as there might be individuals that are instances of $[D]C$ simply because they do not exist in the subcontext designated by D . In fact, it is easy to prove that for any C, D and r the following equivalences hold: $[D]C = \neg D \sqcup \langle D \rangle C$ and $\langle D \rangle C = [D]C \sqcap D$.

3. Reasoning with relative terminologies

In this section we define the problem of satisfiability and discuss some issues concerning computational properties of $\text{DL}^{\mathcal{C}}$. Next, we present two examples embedded in a decidable subset of the language.

3.1. Satisfiability and computational properties

As for most formalisms within the DL paradigm, the basic reasoning service being of interest for $\text{DL}^{\mathcal{C}}$ is *satisfiability checking*. The notion of *satisfaction* of an axiom is relativized here to the context structure and a particular context. For a context structure \mathcal{C} , a context w , and a TBox/ABox axiom α , we say that α is *satisfied* in \mathcal{C} in the context w , or shortly $\mathcal{C}, w \models \alpha$ iff $\mathcal{I}_w \models \alpha$. Consequently, satisfaction of contextualized axioms conservatively extends the definition of satisfaction used in the basic DLs:

- $\mathcal{I}_w \models \diamond \alpha$ iff $\exists w \triangleleft v : \mathcal{I}_v \models \alpha$

- $\mathcal{I}_w \models \Box\alpha$ iff $\forall w \triangleleft v : \mathcal{I}_v \models \alpha$
- $\mathcal{I}_w \models \langle D \rangle \alpha$ iff $\exists w \triangleleft v, \Delta^{\mathcal{I}_v} = D^{\mathcal{I}_w} : \mathcal{I}_v \models \alpha$
- $\mathcal{I}_w \models [D]\alpha$ iff $\forall w \triangleleft v, \Delta^{\mathcal{I}_v} = D^{\mathcal{I}_w} : \mathcal{I}_v \models \alpha$

We say that a knowledge base is satisfied in a context w whenever all its axioms are satisfied in w . Finally, a context structure \mathcal{C} with the top context \hat{w} is a *model* of a knowledge base when all axioms in the knowledge base are satisfied in \hat{w} . Considering the satisfiability conditions and the formal properties of the underlying Kripke frames, we strongly suspect that decidability of the satisfaction problem, and consequently of other standard reasoning problems in $\text{DL}^{\mathcal{C}}$ (e.g. subsumption, instance checking, etc. (Baader et al. 2003)), should be preserved. In the next section we will discuss a syntactic restriction of the language whose decidability we show by a simple argument.

As an interesting consequence of the formulation of the framework, we are able to define the notions of *global* (context-independent) and *local* (context-dependent) terms.

Definition 2 (Globality/locality) A DL term τ is global in a context structure \mathcal{C} iff for every $w, v \in W$ such that $w \triangleleft v$ it holds that:

- if τ is an individual name a then $a^{\mathcal{I}_v} = a^{\mathcal{I}_w}$ iff $a^{\mathcal{I}_w} \in \Delta^{\mathcal{I}_v}$, else $a^{\mathcal{I}_v}$ is unspecified,
- if τ is a concept description C then $C^{\mathcal{I}_v} = C^{\mathcal{I}_w} \cap \Delta^{\mathcal{I}_v}$,
- if τ is a role r then $r^{\mathcal{I}_v} = r^{\mathcal{I}_w} \cap \Delta^{\mathcal{I}_v} \times \Delta^{\mathcal{I}_v}$,

Otherwise, τ is local in \mathcal{C} .

Notably, the dichotomy of *global* vs. *local* terms, in the above sense, follows the distinction between *rigid* and *non-rigid* designators, as they are often denoted in the philosophy of language. Rigid designators are terms which designate the same things in all possible worlds in which those things exist, and do not designate anything in the remaining worlds. Non-rigid designators are exactly those terms which fail to satisfy the same condition. A suitable and explicit selection of assumptions regarding globality/locality of the employed vocabulary is of a great importance from the perspective of reasoning with relative terminologies. On the one hand, the rules of the reasoning calculus should be properly aligned with the modeling intentions with respect to which parts of the represented terminology are actually context-dependent and which are to be interpreted rigidly.¹ On the other one, the choice of assumptions is known to directly affect computational properties of the resulting models, i.e. decidability and complexity of reasoning.²

3.2. Reasoning in $\mathcal{ALC}^{\mathcal{C}}$ – examples

Let us finally present two small examples of (in)valid inferences in the $\text{DL}^{\mathcal{C}}$. To this end we will first specify a small, yet still sufficiently expressive subset of $\text{DL}^{\mathcal{C}}$, which can be easily shown to be decidable. As the basis we will use the DL \mathcal{ALC} , whose concept constructors and their semantics are presented in Table 1. We extend \mathcal{ALC} to $\mathcal{ALC}^{\mathcal{C}}$ by posing the following requirements:

¹E.g. in typical applications ambiguity of individual names is not considered an issue, hence it is natural to impose their rigid interpretation on the level of inference rules.

²Some authors report, for instance, that presence of global roles dramatically increases the complexity of the decision problem (Wolter & Zakharyashev 1999b, Wolter & Zakharyashev 1999a).

$$\begin{aligned}
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\exists r.C)^{\mathcal{I}} &= \{x \mid \exists y (\langle x, y \rangle \in r^{\mathcal{I}} \wedge y \in C^{\mathcal{I}})\} \\
(\forall r.C)^{\mathcal{I}} &= \{x \mid \forall y (\langle x, y \rangle \in r^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}})\}
\end{aligned}$$

Table 1: Concept constructors in the DL \mathcal{ALC}

1. Every axiom in \mathcal{ALC} is a proper axiom in \mathcal{ALC}^c .
2. If C, D are concept descriptions in \mathcal{ALC} , then $\langle D \rangle C$ and $[D]C$ are proper concept descriptions in \mathcal{ALC}^c .
3. If α is a TBox axiom in \mathcal{ALC} , then $\Box \alpha$ is a proper axiom in \mathcal{ALC}^c .
4. No other expressions are allowed in \mathcal{ALC}^c .
5. (Only) individual names are interpreted rigidly in \mathcal{ALC}^c .

The resulting language is decidable. First, observe that we strictly separate TBox axioms containing modalized concept descriptions from the ones bound by the \Box operator. Moreover, note that axioms of the former type might contain only a finite number of $\langle \cdot \rangle$ and $[\cdot]$ operators (while no \diamond nor \Box), where each occurrence of $[D]C$ can be replaced by $\neg D \sqcup \langle C \rangle$ (see Section 2.3). Similarly, the ABox can be reduced to the form in which there is only a finite number of occurrences of $\langle \cdot \rangle$ and no other non- \mathcal{ALC} constructs. Consequently, since every $\langle \cdot \rangle$ uniquely determines the accessible subcontext, it follows that every satisfiable DL^c knowledge base has to be satisfied in a model based on a finite context structure. Thus the problem of checking satisfiability of a DL^c knowledge base can be reduced to the problem of checking satisfiability of a finite number of \mathcal{ALC} knowledge bases, which is of course decidable.³

Example 1 (The biggest city is not a big thing) Consider the scenario presented in Figure 1 from the introductory section. Let us assert that New York is indeed the biggest city and further assume that in every possible context the concept `The_Biggest` is subsumed by `Big`:

1. $\mathcal{A} = \{ \text{New_York} : \langle \text{City} \rangle \text{The_Biggest} \}$
2. $\mathcal{T} = \{ \Box(\text{The_Biggest} \sqsubseteq \text{Big}) \}$

Given no additional knowledge the following statement does not follow:

3. `New_York : Big`

Since New York is the biggest in the context of cities (1) it must be also big in the same context (2). Nevertheless, the interpretation of `Big` in the context of cities is independent

³In (Klarman & Schlobach 2009) we present a tableau-based decision procedure for the same language without TBox axioms bound by the \Box operator. Given the finiteness of the involved context structures, however, presence of these constructs obviously does not affect the complexity of reasoning and can be straightforwardly covered in the calculus.

from its interpretation in other contexts, in particular in the top context, in which our query (3) should be satisfied. Hence New York does not have to be an instance of **Big** in the top context. As an illustration consider the following canonical model invalidating the inference (x is any object different from New York):

$$\begin{aligned}
W &= \{\hat{w}, w\} \\
\triangleleft &= \{\langle \hat{w}, w \rangle\} \\
\Delta &= \{\text{New_York}, x\} \\
\hat{w} &= \Delta^{\mathcal{I}_{\hat{w}}} = \{\text{New_York}, x\} \\
w &= \Delta^{\mathcal{I}_w} = \{\text{New_York}\} \\
\text{City}^{\mathcal{I}_{\hat{w}}} &= \{\text{New_York}\} \\
\text{The_Biggest}^{\mathcal{I}_{\hat{w}}} &= \text{Big}^{\mathcal{I}_{\hat{w}}} = \emptyset \\
\text{The_Biggest}^{\mathcal{I}_w} &= \text{Big}^{\mathcal{I}_w} = \{\text{New_York}\}
\end{aligned}$$

Example 2 (Are tall men tall people?) Consider a simple terminology defining a person as a man or a woman, where the last two are disjoint concepts. Further, we assume the concepts **Tall** and **Short** are globally disjoint, and assert that individual John is tall as compared to men.

1. $\mathcal{T} = \{ \text{Person} \equiv \text{Man} \sqcup \text{Woman}$
2. $\text{Man} \sqcap \text{Woman} \sqsubseteq \perp$
3. $\square(\text{Tall} \sqcap \text{Short} \sqsubseteq \perp) \}$
4. $\mathcal{A} = \{ \text{John} : \langle \text{Man} \rangle \text{Tall} \}$

The following assertion is entailed by the knowledge base:

5. $\text{John} : \langle \text{Person} \sqcap \neg \text{Woman} \rangle \neg \text{Short}$

Notice that since the concept $\text{Person} \sqcap \neg \text{Woman}$ is equivalent to **Man** in the top context (1,2) then obviously both of them designate exactly the same context. Since John is tall in that context (4), and in every context tall objects cannot be short at the same time (3), it follows naturally, that John is in that context also an instance of $\neg \text{Short}$. Observe, however, that it cannot be inferred that John is a non-short person, as nothing is known about the tallness of John in the context of all people.

4. Related work

The language DL^c can be classified as an instance of *modal* or *multi-dimensional* (Wolter & Zakharyashev 1999b, Wolter & Zakharyashev 1999a) DLs, a family of expressive description languages being a fragment of *multi-dimensional modal logics* (Marx & Venema 1997, Kurucz, et al. 2003). To our knowledge, DL^c constitutes a unique proposal explicitly employing this framework for the problem of contextualization of DL knowledge bases, and moreover, it is the only attempt of addressing the specific problem of relativity, i.e. contextualization of DL constructs by comparison classes. The most commonly considered perspectives on contextualization in DLs focus instead on:

1. *Integration of knowledge bases describing local views on a domain* (Bouquet et al. 2003, Benslimane et al. 2006, Borgida & Serafini 2003). In this perspective, one considers a finite set of DL knowledge bases related by bridge rules, of a certain form, which allow for relating concepts belonging to different sources.
2. *Contextualization as levels of abstraction* (Goczyla et al. 2007, Grossi 2007). Within this approach, a knowledge base is modeled as a hierarchical structure organized according to the levels of abstraction over the represented knowledge. The entailments of the higher levels hold in the more specific ones, but not vice versa.

Although the underlying models can be in both cases embedded in two-dimensional Kripke semantics, analogical to ours, the expressive power of the modal machinery is not utilized on the level of language, and thus the formalisms remain strictly less expressive than DL^C . It can be expected, however, that restricted fragments of DL^C and enriched variants of the other approaches to contextualization might coincide on some problems in terms of expressiveness and semantical compatibility.

DL^C shares also some significant similarities with *dynamic epistemic logics*, in particular, with the *public announcement logic* (PAL) (van Ditmarsch, et al. 2007), which studies the dynamics of information flow in epistemic models. Interestingly, our modalities involving named contexts can be seen as public announcements, in the sense used in PAL, whose application results in a dynamic reduction of the description (epistemic) model to only those individuals (epistemic states) that satisfy given description (formula). Unlike in PAL, however, we allow for much deeper revisions of the models, involving also the interpretation function, e.g. it is possible that after contextualizing the representation by $\langle C \rangle$ there are no individuals that are C , simply because C gets essentially reinterpreted in the accessed world.

Finally, we should mention the loosely related problem of vagueness, inherent to the use of relative terms, such as considered in this paper. Traditionally, the problem has been analyzed on the grounds of *fuzzy logics*, which recently have been also successfully coupled with description languages, giving raise to *fuzzy DLs* (Straccia 2005). The ideas underlying fuzzy semantics, however, are orthogonal to the ones motivating our work, and thus can in principle complement each other. While fuzzy logic replaces a binary truth function with a continuous one when defining an interpretation of a relative term, the semantics of DL^C allows for applying a number of truth functions instead of a single one, depending on the context of interpretation. Clearly, none of the two semantics can solve the problems handled by the other, while together they give a very broad account of the problem of relativity of meaning.

5. Conclusions

Motivated by the current challenges of the research on the Semantic Web, we have presented an extension to the standard Description Logics for representing and reasoning with relative terms, i.e. terms whose precise meaning depends on the choice of a particular comparison class. We have argued that the language is powerful enough to capture a number of intuitions associated with the natural use of such terms, and we moreover believe that a thorough investigation of its expressivity should reveal even more interesting applications. Naturally, the gain in expressivity is expected to come at a price of worse

computational properties, the subject which we aim to study as part of the future research. It is likely that in order to achieve an optimal balance it will be necessary to restrict the syntax, possibly down to the most useful modalities \square and $\langle \cdot \rangle$, along the same lines as we have explored in the examples presented in the paper (Section 3.2.).

In principle, a clear advantage of the formalism is its backward compatibility with the standard DL languages. Note that every satisfiable DL knowledge base is at the same time a satisfiable DL^C knowledge base. Also, grounding the approach on multi-dimensional DLs gives good prospects for integrating it with other potential extensions embedded within the same paradigm, which slowly get to attract more attention in the context of the Semantic Web, as potentially useful for numerous representation and reasoning tasks.

Acknowledgements

The author is funded by a Hewlett-Packard Labs Innovation Research Program.

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