Automated Reasoning in Artificial Intelligence:

Introduction to Description Logic

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(*part of the content based on the tutorial by* **Stefan Schlobach**)

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Plan for today

- Description Logic knowledge bases
- Representing knowledge bases in Protégé
- Reasoning tasks and their reduction

$\mathcal{ALC}$: syntax and semantics

Syntax:

- *concept names*: $A, B, C \ldots$; e.g.: $Man, Parent, Car$,
- *role names*: $r, s \ldots$; e.g.: $biggerThan, likes, locatedIn$,
- *concept constructors*: $\top,\ A,\ \neg C,\ C \sqcap D,\ C \sqcup D,\ \exists r.C,\ \forall r.C$,

⋆ *individual names*: $a, b \ldots$; e.g.: $john, europe, snoopy$.

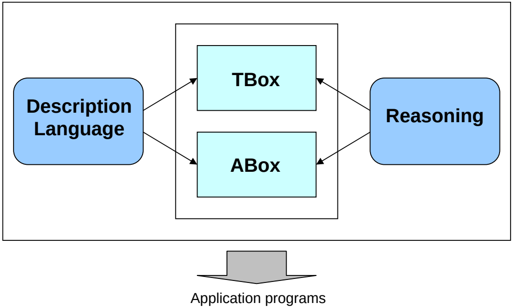
Semantics:

An interpretation is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a non-empty *domain of individuals* and $\cdot^{\mathcal{I}}$ is an *interpretation function*, which maps:

- $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$,
- $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for every concept name A,
- $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for every role name r,
- $\cdot^{\mathcal{I}}$ is extended inductively over complex concepts,

⋆ $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for every individual name a.

DL Knowledge Base



A DL *knowledge base* (alt. *ontology*) $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ consists of:

- *TBox* $\mathcal{T}$, i.e. terminology,
- *ABox* $\mathcal{A}$, i.e. assertions about individuals.

TBox: Syntax

Knowledge about relationships between concepts is expressed by means of *terminological axioms* (TBox axioms):

- *concept inclusion*: $C \sqsubseteq D$
  - necessary conditions for objects of type C.
  - Examples:
    $Elephant \sqsubseteq Animal \sqcap \neg Mouse$
    $Rich \sqcap Famous \sqsubseteq \exists knows.(Rich \sqcap Famous)$
- *concept equivalence*: $C \equiv D$ (short for $C \sqsubseteq D$ and $D \sqsubseteq C$)
  - necessary and sufficient conditions for objects of type C
  - Examples:
    $Animal \sqcap Rational \equiv Man \sqcup Woman$
    $Person \equiv \exists hasParent.Person$

The *TBox* $\mathcal{T}$ of a KB is a finite set of terminological axioms.

Exercise: modeling TBoxes

An artist is someone who created an artwork. A sculpture is an artwork. A painting is an artwork that is not a sculpture. A painter is someone who created a painting. A sculptor is someone who created an artwork and created only sculptures. If an artwork is created by an artist, he has either painted or sculptured it. A multi-talent is both a painter and sculptor.

Model the information as a DL TBox:

Solution:

Exercise: modeling TBoxes

An artist is someone who created an artwork. A sculpture is an artwork. A painting is an artwork that is not a sculpture. A painter is someone who created a painting. A sculptor is someone who created an artwork and created only sculptures. If an artwork is created by an artist, he has either painted or sculptured it. A multi-talent is both a painter and sculptor.

Model the information as a DL TBox:

Solution:

$$
\begin{aligned}
Artist &\equiv \exists created.Artwork \\
Sculpture &\sqsubseteq Artwork \\
Painting &\equiv Artwork \sqcap \neg Sculpture \\
Painter &\equiv \exists created.Painting \\
Sculptor &\equiv \exists created.Artwork \sqcap \forall created.Sculpture \\
Artwork &\sqsubseteq \exists painted\_by.Artist \sqcup \exists sculptured\_by.Artist \\
Multitalent &\sqsubseteq Painter \sqcap Sculptor
\end{aligned}
$$

ABox: Syntax

Knowledge about individuals in the domain expressed in terms of the vocabulary is specified by means of *assertional axioms* (ABox axioms):

- *concept assertion:* $a : C$
  - individual a is an instance of concept C
  - Example:
    $mary : Mother$
    $john : Rich \sqcup \exists hasParent.Rich$
- *role assertions:* $(a, b) : r$
  - individual a is related to b through the role r
  - Example:
    $(john, mary) : likes$
    $(new\_york, amsterdam) : biggerThan$

The *ABox* $\mathcal{A}$ of a KB is a finite set of assertional axioms.

Exercise: modeling ABoxes

Rembrandt created the artwork: "nightwatch", but never created a sculpture. "nightwatch" is a painting. Michelangelo created at least one sculpture.

Model the information as a DL ABox.

Solution:

Exercise: modeling ABoxes

Rembrandt created the artwork: "nightwatch", but never created a sculpture. "nightwatch" is a painting. Michelangelo created at least one sculpture.

Model the information as a DL ABox.

Solution:

$$(rembrandt, nightwatch) : created$$

$$rembrandt : \neg\exists created.Sculpture$$

$$nightwatch : Painting$$

$$michelangelo : \exists created.Sculpture$$

Protégé

Protégé is an *ontology editor* for OWL. But since OWL is a syntactic variant of DLs, OWL ontologies can be seen as DL knowledge bases.

DL vs. Protégé interface:

- OWL nomenclature: concept $\rightsquigarrow$ class, role $\rightsquigarrow$ object property.
- Protégé user-friendly syntax:

$\top \rightsquigarrow$ Thing	$C \sqcap D \rightsquigarrow$ (C and D)	$\exists r.C \rightsquigarrow$ (r some C)
$\bot \rightsquigarrow$ Nothing	$C \sqcup D \rightsquigarrow$ (C or D)	$\forall r.C \rightsquigarrow$ (r only C)
$\neg C \rightsquigarrow$ not C		

- Designated fields in a template for entering axioms:

$C \sqsubseteq D \rightsquigarrow$ Classes / "C" / Superclasses / "D"
$C \equiv D \rightsquigarrow$ Classes / "C" / Equivalent classes / "D"
$a : C \rightsquigarrow$ Individuals / "a" / Types / "C"
$(a, b) : r \rightsquigarrow$ Individuals / "a" / Object prop. asser. / "r" | "b"

Exercise: modeling ontology in Protégé:

Enter the following KB into Protégé:

$$
\begin{aligned}
Artist &\equiv \exists created.Artwork \\
Sculpture &\sqsubseteq Artwork \\
Painting &\equiv Artwork \sqcap \neg Sculpture \\
Painter &\equiv \exists created.Painting \\
Sculptor &\equiv \exists created.\top \sqcap \forall created.Sculpture \\
Multitalent &\sqsubseteq Painter \sqcap Sculptor
\end{aligned}
$$

$(rembrandt, nightwatch) : created$

$rembrandt : \neg\exists created.Sculpture$

$nightwatch : Artwork$

$michelangelo : \exists created.Sculpture$

TBox: Semantics

Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ be an interpretation. $\mathcal{I}$ *satisfies* a terminological axiom in either of the two cases:

- for $C \sqsubseteq D$ if and only $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- for $C \equiv D$ if and only $C^{\mathcal{I}} = D^{\mathcal{I}}$

An interpretation $\mathcal{I}$ is a *model* of the TBox $\mathcal{T}$ *iff* it satisfies every terminological axiom in $\mathcal{T}$.

TBox: Semantics example

Let $\mathcal{I}$ be defined as:

- $\Delta^{\mathcal{I}} = \{rembrandt, michelangelo, rodin, nightwatch, david, sixtChappel, thinker\}$
- $$\begin{aligned} Artwork^{\mathcal{I}} = & \{nightwatch, sixtChappel, thinker, david\}, \\ Sculptor^{\mathcal{I}} = & \{rodin, michelangelo\} \\ Sculpture^{\mathcal{I}} = & \{thinker, david\} \\ Painter^{\mathcal{I}} = & \{rembrandt, michelangelo\} \\ Painting^{\mathcal{I}} = & \{nightwatch, sixtChappel\} \\ sculptured^{\mathcal{I}} = & \{(rodin, thinker), (michelangelo, david\} \\ created^{\mathcal{I}} = & \{(rembrandt, nightwatch), (michelangelo, sixtChappel), \\ & (michelangelo, david), (rodin, thinker)\} \end{aligned}$$

Is $\mathcal{I}$ a model of $\mathcal{T}$?

$$\begin{aligned} Painting & \sqsubseteq Artwork \sqcap \neg Sculpture \\ Painter & \equiv \exists created.Painting \\ Sculptor & \equiv \exists sculptured.Artwork \sqcap \forall created.Sculpture \end{aligned}$$

Reasoning tasks for TBoxes

For a TBox $\mathcal{T}$ and concepts C, D occurring in $\mathcal{T}$.

concept satisfiability:

C is *satisfiable* w.r.t. $\mathcal{T}$ *iff* there is a model $\mathcal{I}$ of $\mathcal{T}$: $C^{\mathcal{I}} \neq \emptyset$

subsumption: $\mathcal{T} \models C \sqsubseteq D$?

C is *subsumed* by D in $\mathcal{T}$ *iff* $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in every model $\mathcal{I}$ of $\mathcal{T}$

equivalence: $\mathcal{T} \models C \equiv D$?

Concepts C and D are *equivalent* in $\mathcal{T}$ *iff* $C^{\mathcal{I}} = D^{\mathcal{I}}$ in every model $\mathcal{I}$ of $\mathcal{T}$.

Reduction of TBox reasoning tasks

All TBox problems in $\mathcal{ALC}$ are reducible to concept satisfiability:

- C is *subsumed by* D in $\mathcal{T}$ $\Leftrightarrow$ $C \sqcap \neg D$ is *unsatisfiable* w.r.t. $\mathcal{T}$

  Proof: C is subsumed by D in $\mathcal{T}$
  $\Leftrightarrow$ for every model $\mathcal{I}$ of $\mathcal{T}$ it holds that $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
  $\Leftrightarrow$ for every model $\mathcal{I}$ of $\mathcal{T}$ it holds that $C^{\mathcal{I}} \cap (\neg D)^{\mathcal{I}} = \emptyset$
  $\Leftrightarrow$ there is no model $\mathcal{I}$ of $\mathcal{T}$ s.t. $(C \sqcap \neg D)^{\mathcal{I}} \neq \emptyset$
  $\Leftrightarrow$ $C \sqcap \neg D$ is unsatisfiable w.r.t. $\mathcal{T}$.

- C and D are *equivalent* in $\mathcal{T}$ $\Leftrightarrow$ C is *subsumed by* D in $\mathcal{T}$ and D is *subsumed by* C in $\mathcal{T}$

Note: Notice that reduction to concept satisfiability requires: $\sqcap$ and $\neg$ for complex concepts.

ABox: Semantics

Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ be an interpretation. $\mathcal{I}$ *satisfies* an assertional axiom in either of the two cases:

- for $a : C$ if and only $a^{\mathcal{I}} \in C^{\mathcal{I}}$
- for $(a, b) : r$ if and only $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$

An interpretation $\mathcal{I}$ is a *model* of the ABox $\mathcal{A}$ *iff* it satisfies every assertional axiom in $\mathcal{A}$.

An interpretation $\mathcal{I}$ is a *model* of $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ *iff* $\mathcal{I}$ is a *model* of both $\mathcal{A}$ and $\mathcal{T}$.

Note: By default *Unique Name Assumption* applies in DLs (but not in OWL!)

ABox: Semantics example

Let $\mathcal{I}$ be defined as:

- $\Delta^{\mathcal{I}} = \{rembrandt, michelangelo, rodin, nightwatch, david, sixtChappel, thinker\}$
- $Artwork^{\mathcal{I}} = \{nightwatch, sixtChappel, thinker, david\}$,
  $Sculpture^{\mathcal{I}} = \{thinker, david\}$
  $Painting^{\mathcal{I}} = \{nightwatch, sixtChappel\}$
  $created^{\mathcal{I}} = \{(rembrandt, nightwatch), (michelangelo, sixtChappel), (michelangelo, david), (rodin, thinker)\}$

Is $\mathcal{I}$ a model of $\mathcal{A}$?

$$(rodin, thinker) : created$$

$$nightwatch : Artwork$$

$$rembrandt : \neg\exists created.Sculpture$$

Reasoning tasks for ABoxes

For a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, a concept C, a role r and individuals a, b:

ABox consistency:

$\mathcal{A}$ is *consistent* w.r.t $\mathcal{T}$ *iff* there is a model of $\mathcal{K}$.

Note: in such case we also say that $\mathcal{K}$ is *satisfiable*.

instance checking: $\mathcal{K} \models a : C$? (resp. $\mathcal{K} \models (a, b) : r$) ?

- a is *an instance* of C in $\mathcal{K}$ *iff* every model of $\mathcal{K}$ is a model of $a : C$
- (a, b) are *in related* r in $\mathcal{K}$ *iff* every model of $\mathcal{K}$ is a model of $(a, b) : r$

Derived tasks:

- *retrieval:* Given a concept C and an Abox $\mathcal{A}$ find all individuals a such that $\mathcal{K} \models a : C$
- *realization:* Given an individual a and a set of concepts, find the most specific concept C such that $\mathcal{K} \models a : C$.

Reduction of ABox reasoning tasks

All ABox problems in $\mathcal{ALC}$ are reducible to ABox consistency. For KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$:

- a is *an instance of* C in $\mathcal{K} \Leftrightarrow \mathcal{A} \cup \{a : \neg C\}$ is *inconsistent* w.r.t. $\mathcal{T}$.

  Proof: a is an instance of C in $\mathcal{K}$
  $\Leftrightarrow$ for every model $\mathcal{I}$ of $\mathcal{K}$ it holds that $a^{\mathcal{I}} \in C^{\mathcal{I}}$
  $\Leftrightarrow$ there is no model $\mathcal{I}$ of $\mathcal{K}$ s.t. $a^{\mathcal{I}} \in (\neg C)^{\mathcal{I}}$
  $\Leftrightarrow$ there is no model $\mathcal{I}$ of both $\mathcal{A} \cup \{a : \neg C\}$ and $\mathcal{T}$
  $\Leftrightarrow$ $\mathcal{A} \cup \{a : \neg C\}$ is inconsistent w.r.t. $\mathcal{T}$.

- (a, b) are *in relation* $r \Leftrightarrow (a, b) : r \in \mathcal{A}$.
- *retrieval* and *realization* equivalent to a finite number of instance checking and subsumption tasks.

Reduction of reasoning tasks

...and finally:

- C is *satisfiable* w.r.t. $\mathcal{T} \Leftrightarrow \mathcal{A} = \{a : C\}$ is *consistent* w.r.t. $\mathcal{T}$, for a fresh individual name a

  Proof: C is satisfiable w.r.t. $\mathcal{T}$
  $\Leftrightarrow$ there is a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}} \neq \emptyset$
  $\Leftrightarrow$ there is at least one instance of C in $\mathcal{I}$ — name it a
  $\Leftrightarrow$ there is a model of $\mathcal{T}$ which satisfies assertion $a : C$
  $\Leftrightarrow$ there is a model of $\mathcal{T}$ which is a model of the ABox $\mathcal{A} = \{a : C\}$
  $\Leftrightarrow$ $\mathcal{A} = \{a : C\}$ is consistent w.r.t. $\mathcal{T}$, for a fresh name a.

Hence:

- All reasoning tasks in $\mathcal{ALC}$ can be reduced to a single task of checking ABox consistency w.r.t. TBox.
- The complexity of ABox consistency checking cannot be lower than that of the other tasks.

Summary

- A DL *knowledge base* $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ consists of the TBox (terminology) $\mathcal{T}$ and the ABox (assertions) $\mathcal{A}$.
- Axioms of a KB *restrict the possible models*.
- The reasoning tasks in $\mathcal{ALC}$ for TBoxes and ABoxes can be reduced to checking *ABox consistency w.r.t. TBox*.

Next:

- Tableau algorithm for reasoning in $\mathcal{ALC}$.